

Super-resolving multiresolution images with band-independant geometry of multispectral pixels

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Abstract—A new resolution enhancement method is presented for multispectral and multi-resolution images, such as these provided by the Sentinel-2 satellites. Starting from the highest resolution bands, band-dependent information (reflectance) is separated from information that is common to all bands (geometry of scene elements). This model is then applied to unmix low-resolution bands, preserving their reflectance, while propagating band-independent information to preserve the sub-pixel details. A reference implementation is provided, with an application example for super-resolving Sentinel-2 data.

I. INTRODUCTION

Earth Observation missions typically operate at medium to low resolution ranges in order to favor both larger satellite swath and better temporal revisit of the same site (e.g. 3-4 days over Europe for the Sentinel-2 Satellite series). For each acquisition, optical constraints furthermore often restrict that only some spectral bands have maximal resolution. For example, a common case is to compensate the smaller pixel size of the higher spatial resolution bands (e.g. 10m/pixel) by capturing light over a larger spectrum range (e.g. 4 large bands in the red, green, blue and near infrared for Sentinel-2). The limit case being a single high-resolution panchromatic band (Pleiades, Spot, Landsat...). Narrower spectral bands, invaluable for specific measurements (e.g. chlorophyll or water vapor absorption wavelengths) are then provided at lower resolution (e.g. 20m/pixel and 60m/pixel for Sentinel-2). Yet, this trade-off on spectral and spatial resolutions may become a limiting factor for many Earth Observation applications, for example for getting accurate land cover classification at the highest resolution [1]. Some techniques have thus been devised in order to propagate the high-resolution spatial details to the lower-resolution dedicated bands while preserving their spectral content. These can be sorted along the following categories:

- Panchromatic sharpening [2]: Using a very wide band with high-resolution in order to compensate the lower resolution of narrow bands. Multiple variants exist, from a simple renormalization of the multispectral bands [3] to more advanced unmixing techniques which estimate the contribution of each spectral band to the panchromatic one [4], possibly with pre-processing steps designed to uncorrelate each component [5], [2]. Limits of this technique are when the panchromatic band does not cover all narrow spectral bands, which could result in color distortion for the uncovered bands. The advantages of panchromatic sharpening are its efficiency, and its applicability even when only a single high-resolution band is acquired.

- Probabilistic [6], [7]: The spectral information in each sub-pixel of an original low-resolution pixel is determined by maximizing a probabilistic model, constrained by the observed data at all bands and resolutions (possibly including the panchromatic band). A Bayesian formulation can be chosen to represent this constraint which, provided this does not becomes intractable, allows hyperparameters to be set according to prior knowledge.

- Sensor-based: If the sensor has a known point spread function (PSF), then deconvoluting it enhances the resolution of the acquired images [8]. But the PSF for many satellites can only be estimated empirically (Sentinel-2, Spot-5, Landsat-8 [1]). When that is the case, limits on sub-pixel detection can be established [1].

- Learning-based: These methods exploit local patterns in the low-resolution images, and propagate these features (e.g. edges) to infer the higher resolution image [9]. Many models may be used to “learn” the features: neural network [10], example-based [11] with kernel ridge regression [12], deep learning [13] and more references therein, including for cross-image learning. These methods can be applied on a single resolution image, possible with different channels (typically red, green, blue [13]). Filling details from learned (or duplicated) texture features might be very good to produce visually plausible results [11]. Their main problem, similar to in-painting with image-based examples [14], is that “hallucinated” [9] details do not necessarily correspond to true higher-resolution objects (esp. with non-local or cross-image features) and then become misleading pixels for land monitoring purposes.

- Scaling laws: Instead of learning local patterns, this method learns multi-scale relationships in the data such as local power laws between spatial extent and band values [17]. Such power laws are inferred from data above the acquisition resolution but, assuming the same laws remain valid below that resolution, these can then be used to infer sub-pixels of the original image. Very good results have been obtained with such methods for turbulent oceanic data [18], where energy cascades translates to power laws spanning multiple decades. However, for land monitoring purposes, usually no such physical interpretation can be found: for example a mixture of trees, houses and roads in a peri-urban environment is not locally scale-invariant. Similarly, when working in the frequency domain, methods based on wavelet decompositions and upsampling with a smooth filter (e.g. bicubic) usually fail for a similar reason: knowledge that there is a tree (or road, house...) a few hundred meters away (i.e. the wavelet support size) does not help subdivide a local pixel into its higher-resolution components.

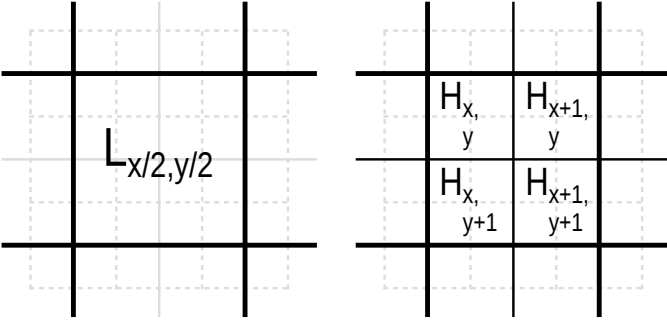


Figure 1. Introducing the indexing and the relations between the low and high resolution pixels, for the simple case of doubling the resolution. The grayed lines indicate boundaries from Fig. 2, for ease of interpretation.

The method presented below works with only local information. It relies on the observation that the proportion of objects of the same nature within a pixel area (e.g. 30% urban area, 70% trees), is a physical property of that pixel and therefore independent of the spectral band. Only the reflectance of these objects may change from band to band. Moreover, there is no reason why pixel boundaries would match natural object boundaries. The method thus identifies generic “shared” information between adjacent pixels, then commonalize the geometric aspects of these shared values across bands. High resolution bands are used to separate band-dependant reflectance. The geometric information is then used to unmix the low-resolution pixels, while preserving their overall reflectance.

Section 2 presents the super-resolution problem and Section 3 details how that problem is addressed by the model introduced in this paper. Section 4 shows super-resolution results and Section 5 demonstrates the influence of each step of the algorithm.

II. PROBLEM DESCRIPTION

A. Super-resolution formulation

Let L be an observed low resolution image with $N_x/2$ columns and $N_y/2$ rows. We consider the problem of finding a high resolution image H with N_x columns and N_y rows. Each low resolution pixel $L_{x/2,y/2}$ thus corresponds to 4 high resolution pixels, as depicted in Fig. 1. Averaging these pixels should give the original observed low-resolution pixel back:

$$L_{x/2,y/2} = \frac{1}{4} (H_{x,y} + H_{x+1,y} + H_{x,y+1} + H_{x+1,y+1}) \quad (1)$$

Images remotely sensed from satellites are subject to multi-scale transforms (including atmospheric corrections [19]) before being released as a useable product. These transforms are out of scope of the present document but may introduce correlations between high-resolution pixels (e.g. due to scattering), hence should be applied before (or integrated to) super-resolution. Similarly, known PSF [8] should be deconvoluted in addition to the method presented below. In any case, Eq. 1 ensures that down-sampling by averaging the high-resolution solution will recover the observations.

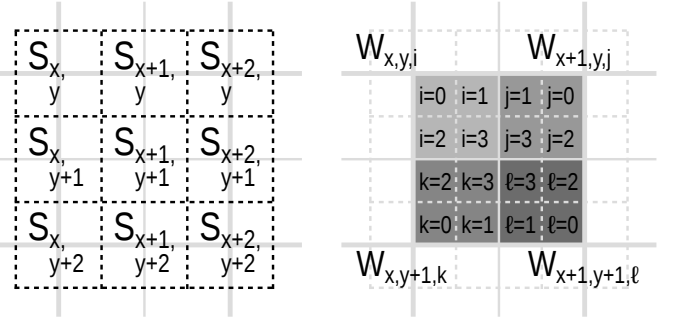


Figure 2. Values S shared between neighbor pixels, and how these are combined by weights W to form the high-resolution pixels. Compare with the boundaries from Fig. 1, indicated as grayed lines for ease of interpretation. Shared values span over multiple pixels by definition, they represent the reflectance of elements that are common to these pixels. Weights are internal to each pixel, they represent the proportion of these elements within the pixel and thus independant of the spectral band.

Eq. 1 is undetermined, with 3 free parameters per low resolution pixel. Some extra constraints are needed, which are extracted from available high-resolution data bands.

B. Shared information between neighbor pixels

Natural objects do not fall exactly on pixel boundaries. Therefore, some content is shared between nearby pixel values. This shared information is explicitly defined as in Fig. 2, left. For example, $S_{x+1,y+1}$ is the reflectance corresponding to the shared part between high-resolution pixels $H_{x,y}$, $H_{x+1,y}$, $H_{x,y+1}$ and $H_{x+1,y+1}$ (compare Fig. 1 and Fig. 2). This particular element is fully within the observed low-resolution pixel $L_{x/2,y/2}$. Other shared values may span multiple low-resolution pixels. With this notation, there are $(N_x + 1) \times (N_y + 1)$ spatially shared values S . These located at the image boundaries, or which would span invalid pixels such as in the case of sensor failure, are simply not commonalized and effectively remain internal to the valid pixels. All shared values are expressed in reflectance units and constrained to the range of their respective band.

In remote sensing, the reflectance of each pixel is often considered to be a linear mix [20] of the reflectances of its constituents (e.g. a mix of vegetation and soil). Assuming the shared values correspond to unknown constituents spanning over pixel boundaries, and assuming this linear mixing model, the proportion of each shared value that is present in each pixel is thus determined by weights that are specific to that pixel (Fig. 2, right). This leads to the following mixing equation for the shared values and the weights:

$$H_{x,y} = W_{x,y,0}S_{x,y} + W_{x,y,1}S_{x+1,y} + W_{x,y,2}S_{x,y+1} + W_{x,y,3}S_{x+1,y+1} \quad (2)$$

With the following constraints:

$$\sum_{k=0}^3 W_{x,y,k} = 1 \quad (3)$$

$$W_{x,y,k} \geq 0 \quad \forall k \in [0 \dots 3] \quad (4)$$

III. SOLVING THE SUPER-RESOLUTION PROBLEM

A. Separating band-specific information from information common to all bands

The proportion of mixed elements within a pixel (e.g. 20% road / 80% vegetation) is a physical property of that pixel, but the reflectance of each element depends on the spectral band at which it is observed. Therefore, the weights are common to all bands, while shared values are band-dependant. Weights encode the geometric consistency of pixels across bands. Shared values encode the spatial consistency of nearby pixels. The high-resolution data are used to fit the full mixing model, containing both weights and shared values. This step is presented below. The next section addresses how to un-mix low-resolution bands in order to produce the super-resolution result, reusing the weights fit from the high-resolution bands.

Starting from an observed high-resolution band H^o , a down-sampled version L^d of the data is created with Eq. 1. The best mixing model is estimated by minimizing the difference between: a) the observed pixel values H^o , and b) the resolution-enhanced values H^r computed from the down-sampled data L^d . Let us subscript data specific to each band with an additional index. Thus, L^d , H^o , H^r and S are subscripted, but not the weights W . Solving this first problem is a constrained minimization, for $k = 0 \dots 3$, and $\beta \in \mathcal{H}$ the set of high-resolution bands:

$$\{S^{opt}, W^{opt}\} = \operatorname{argmin} \sum_{\beta \in \mathcal{H}} \sum_{x,y} \|H_{\beta,x,y}^o - H_{\beta,x,y}^r\|^2 \quad (5)$$

with each $H_{\beta,x,y}^r$ term given by Eq. 2. An iterative solver [21] is used for constrained least squared error optimization, allowing Eq. 3 to be implemented by a reparametrization and Eq. 4 by soft boundaries (a reference implementation is provided, link given in appendix). Initial weights for the iterative algorithm are set to $\frac{1}{4}$ (i.e. equal influence to all shared values, see Fig. 2, left). The initial shared values S_{β}^{ini} are computed by averaging each high-resolution pixel H_{β}^o that partially covers S_{β}^{ini} in Fig. 2. For example, $S_{\beta}^{ini} = \frac{1}{2} \left(H_{\beta,x/2,y/2}^o + H_{\beta,x/2+1,y/2}^o \right)$ for $x < N_x - 1$ and $y < N_y$.

At the end of this step, both shared values S_{β}^{opt} between high-resolution pixels, and weights W^{opt} common to all bands, are available.

B. Estimating shared values from low-resolution data

Shared values S^{opt} are found by optimization on high-resolution data, so they cannot be estimated directly on the low-resolution bands with the above procedure. Instead, the relation between S^{opt} and nearby low-resolution pixels can be learned from downsampled high-resolution bands L^d . That relation is also expressed as a geometric property common to all bands, so it can be used in order to produce a first estimate S^{fit} for the low-resolution bands. More specifically, a second set of mixing coefficients V is built in order to fit $S_{\beta,x,y}^{opt}$ from low-resolution pixels $L_{\beta,n}^d$ at nearby locations $n \in \mathcal{N}(x,y)$. See Fig. 4, with \mathcal{N} being either the corner, middle or inner

variant depending on the position of (x,y) with respect to the low-resolution reference pixel.

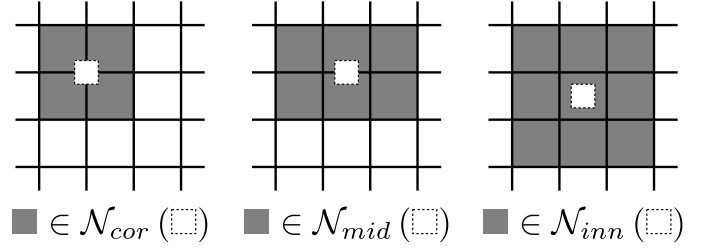


Figure 4. Low-resolution neighborhoods for high-resolution shared values. Depending on the location of the shared value with respect to the center reference pixel (corner, middle, inner), the neighborhood consists of either 4, 6 or 9 low-resolution pixel locations. Other corner and middle locations are deduced by a rotation of the pattern.

These neighborhoods hopefully capture local features (e.g. edges), in the form of up to 9 coefficients $v_{x,y,n}$ for each high-resolution pixel (x,y) in the image:

$$V = \operatorname{argmin} \sum_{\beta \in \mathcal{H}} \sum_{x,y} \left\| S_{\beta,x,y}^{opt} - \sum_{n \in \mathcal{N}(x,y)} v_{x,y,n} L_{\beta,n}^d \right\|^2 \quad (6)$$

With this global optimization, the set of coefficients V encodes how the shared values are related to their low-resolution neighborhood, independently of the spectral band. They are fit from the high-resolution bands, and then propagated to the low-resolution bands $b \in \mathcal{L}$ in order to provide a first estimate S^{fit} for the shared values in each band b :

$$S_{b,x,y}^{fit} = \sum_{n \in \mathcal{N}(x,y)} v_{x,y,n} L_{b,n} \quad (7)$$

The fit from Eq. 6 is rarely perfect and values $S_{\beta,x,y}^{fit} = \sum_{n \in \mathcal{N}(x,y)} v_n L_{\beta,n}^d$ can also be computed for the original high-resolution bands. The ratios $q_{\beta,x,y} = S_{\beta,x,y}^{opt} / S_{\beta,x,y}^{fit}$ are then exploited in order to mimic the panchromatic sharpening method in [3], but using the multiple high-resolution bands instead. For Sentinel-2, no panchromatic band is available to encompass the spectrum of all low-resolution bands. An idea is to empirically replace the panchromatic band by a combination of high-resolution bands that yield close spectral responses for the shared values. For each low resolution band $b \in \mathcal{L}$, and for each shared value, a normalized proximity measure is defined as $p_{b,\beta,x,y} = |S_{b,x,y}^{fit} - S_{\beta,x,y}^{fit}| / \max_{\alpha} |S_{b,x,y}^{fit} - S_{\alpha,x,y}^{fit}|$, where the normalization is performed by using the maximum discrepancy over all high resolution bands. Combining the high-resolution sharpening ratios is then performed by geometric averaging, using these proximity measures as weighting factors:

$$\bar{q}_{b,x,y} = \exp \left(\left(\sum_{\beta} p_{b,\beta,x,y} \log q_{\beta,x,y} \right) / \sum_{\beta} p_{b,\beta,x,y} \right) \quad (8)$$

This overall average sharpening ratio is used as a prefactor for setting corrected shared values $S_{b,x,y}^{cor} = \bar{q}_{b,x,y} S_{b,x,y}^{fit}$ for the low resolution bands. Having now estimated high-resolution

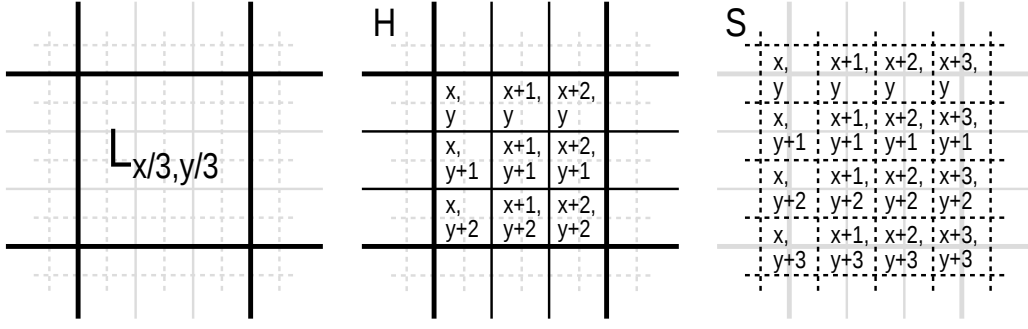


Figure 3. Introducing the indexing and the relations between the low and high resolution pixels, for the case of tripling the resolution. The shared values between pixels are also indicated. Weights are still internal to each high-resolution pixel with exactly the same structure as in Fig. 2, right.

shared values for the low-resolution bands, it is a simple matter of combining these $S_{b,x,y}^{cor}$ with the weights W by applying Eq. 2, in order to produce super-resolved pixels. A final rescaling of these $H_{b,x,y}^r$ is performed so as to ensure reflectance is preserved (Eq. 1).

C. Super-resolving 60m/pixel bands

In this setup, each low-resolution pixel corresponds to 36 values at 10m/pixel. Solving this directly is not tractable, but an indirect solution with an intermediate step is feasible:

- In a first pass, a 60m/pixel band is super-resolved to 20m/pixel. There are then 9 sub-pixels to infer for each low-resolution pixel, with 8 free parameters (Fig. 3 and Eq. 9 below). However, there are also 10 bands at 20m/pixel (including the 4 bands at 10m/pixel, downsampled). These provide enough constraints for the inference of weight values, common to all these bands, computed with a modified version of the method presented below.

- In a second pass, the 20m/pixel solution from the first pass is super-resolved down to 10m/pixel with the method described in the previous sections.

Adapting the above notation to the 9 sub-pixels problem, in this section the low resolution bands are $b \in \mathcal{L}$ at 60m/pixel, while the high resolution bands $\beta \in \mathcal{H}$ consist of the 20m/pixel bands (either original or down-sampled from 10m/pixel). Preserving the reflectance imposes (Fig. 3):

$$L_{b,x/3,y/3} = \frac{1}{9} \sum_{i,j=0}^2 H_{b,x+i,y+j}^r \quad (9)$$

H_b^r is the intermediate super-resolution solution at 20m/pixel for band b . Fig. 3 also shows the relation between L_b , the high-resolution pixels H_b^r and the shared values. With this convention, the weights W follow exactly the same (x, y) pattern with respect to S and H as in Fig. 2, right. The first step of the method, the estimation of both W and S with all available high-resolution bands, is thus also the same as above. All 10 bands $\beta \in \mathcal{H}$ are used as constraints for Eq. 5.

A difference lies in estimating V from nearby pixels. There are still four neighborhood patterns of the “corner” type (see Fig. 4), for shared values $S_{x,y}$, $S_{x+3,y}$, $S_{x,y+3}$ and $S_{x+3,y+3}$. But there are now two “middle” neighborhood patterns for each side of the lower-resolution pixel (e.g. $S_{x+1,y}$, $S_{x+2,y}$),

as well as four “inner” neighborhood patterns instead of one (see Fig. 3, right). With these definitions, Eq. 6 is solved as before. Averaging the sharpening ratios now involves all 10 bands $\beta \in \mathcal{H}$ (instead of 4 in the previous section), but this does not change Eq. 8. Thus, solving the 60m→20m super-resolution problem is performed with very little adaptation.

Once computed, the intermediate solutions at 20m/pixel are further processed by a final 20m→10m super-resolution step, as described in the above sections.

IV. RESULTS

The delta of the Eyre river (France) was selected as it shows a mixture of urban areas, forests, fields, rivers and coastal shallow waters. The image was acquired by the Sentinel 2A satellite on 2016/08/22 and processed with the “sen2cor” atmospheric correction utility [19]. Fig. 5 shows the selected region as a composite image from the 10m/pixel visible bands, where each blue, green, red component was scaled between 1% and 99% of the original reflectance. The infrared band at 10m is also shown as a reference.

The original band 5 (red-edge, 705nm) and its super-resolved version are presented in Fig. 6 as an example of 20m/pixel → 10m/pixel enhancement. Figs. 7 and 8 respectively show the 60m bands B9 (infrared, 945nm) and B1 (violet, 443nm) together with their 10m super-resolved versions. Details not present in the original bands are immediately visible. These correspond to the band-independent information that was extracted from the other bands, and propagated to these images. Although bands 1 and 9 present very different reflectance properties, the exact same weights and geometric information extracted from Eqs. 5,6,8 were applied to both bands. This example demonstrates how the method correctly extracts band-independent information that encodes image details, while preserving the reflectance of each band (Eqs. 1,9).

V. DISCUSSION

The results presented in Figs. 6, 7, 8 are visually pleasing, but it is difficult to assert quantitatively the performance of the super-resolution algorithm on these images, at least not with another source that would provide the ground truth for the super-resolved bands. A common workaround is to first degrade the image, apply a super-resolution algorithm on the downsampled version, and then estimate the difference from

the original with a root mean squared error (MSE) (or the peak signal-to-noise ratio which is just MSE on a log scale). That method may be good for monochromatic single-resolution images [13], but it is not adapted in our multispectral case where geometric details are extracted from other bands. Furthermore, using subsampled bands in order to compute the MSE is already included as part of the current method, in both data values space (Eq. 5) and in spatially shared values space (Eq. 6), and that step is further complemented by \bar{q} ratio sharpening. MSE alone cannot thus be used to quantify the accuracy of the super-resolution.

What can be done, however, is to study the impact of each step of the method. For instance, why not apply ratio sharpening (Eq. 8) directly on data values (instead of spatially shared values) so as to simulate panchromatic sharpening? This would simplify the method drastically. The result of this experiment is shown in Fig. 9, left, to be compared with the correct super-resolved result in Fig. 8, right. The role of spatial consistency is immediately highlighted: without the shared values trick, unacceptable pixel blocks are clearly visible in the result image.

Conversely, if, as explained above, MSE optimization is

already included as part of the method, then why is \bar{q} ratio sharpening useful? Fig. 9, right shows that it is in fact quite important for recovering the fine structures. Given that importance, one may then question the usefulness of extracting weights W as band-independent information, especially since we also compute reverse weights V in a second step. Why would these W encode image details? Fig. 10 shows the result of simply setting these weights to $\frac{1}{4}$ and S^{opt} to the average values, as described in Section III-A, while maintaining \bar{q} ratio sharpening. As expected, details are also smoothed out compared to Fig. 8, right: weights W are defined between high-resolution pixels, hence encode high-resolution details. The reverse weights V encode larger range patterns present in surrounding low-resolution pixels.

Results presented in this section use the 60m/pixel band 1, for which two super-resolution steps are applied. This choice was made so as to enhance and clearly highlight the influence of shared values S , of \bar{q} ratio sharpening, and of weights W , V , on the final result. For 20m/pixel bands only one super-resolution step is applied, but all parts of the method are still needed for good results.

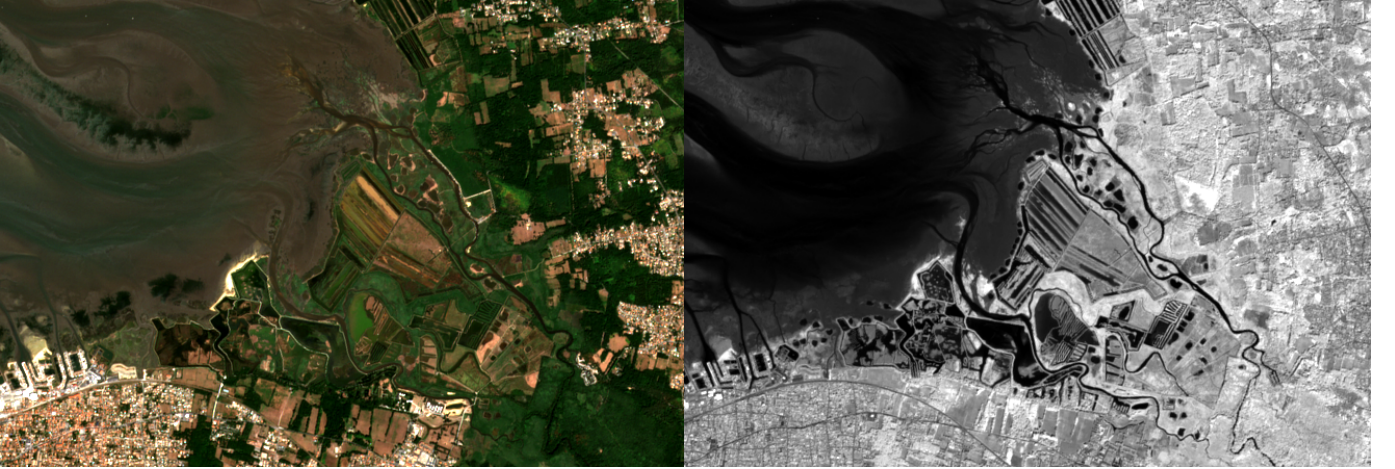


Figure 5. Left: Composite image with original red, green, blue bands at 10m/pixel. Right: Original band 8 (infrared, 842nm) at 10m/pixel.

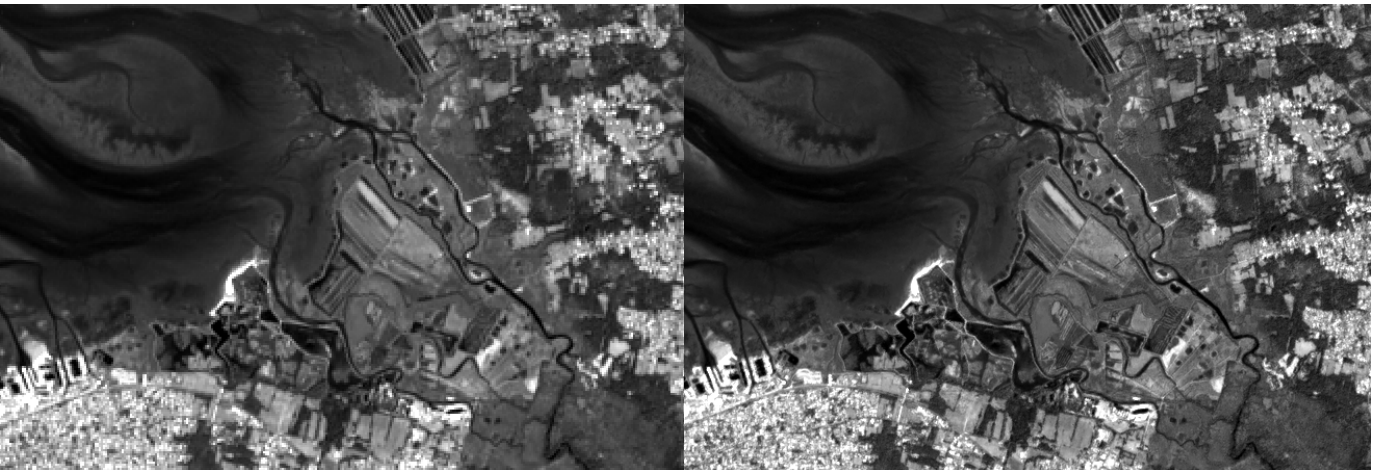


Figure 6. Left: Original band 5 (red-edge, 705nm) at 20m/pixel. Right: Super-resolved band 5 at 10m/pixel.

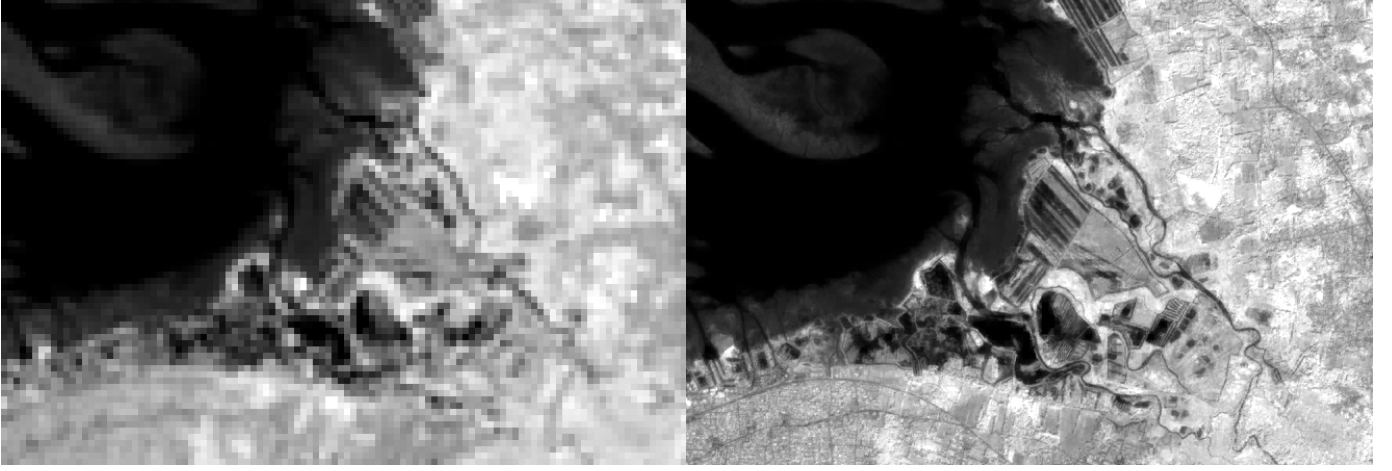


Figure 7. Left: Original band 9 (infrared, 945nm) at 60m/pixel. Right: Super-resolved band 9 at 10m/pixel.

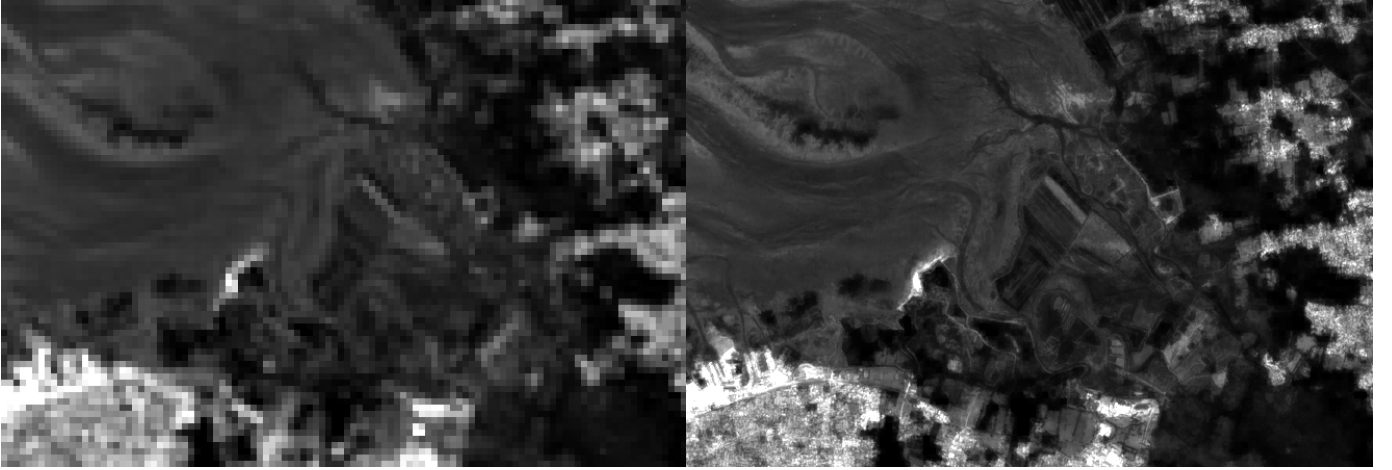


Figure 8. Left: Original band 1 (violet, 443nm) at 60m/pixel. Right: Super-resolved band 1 at 10m/pixel.

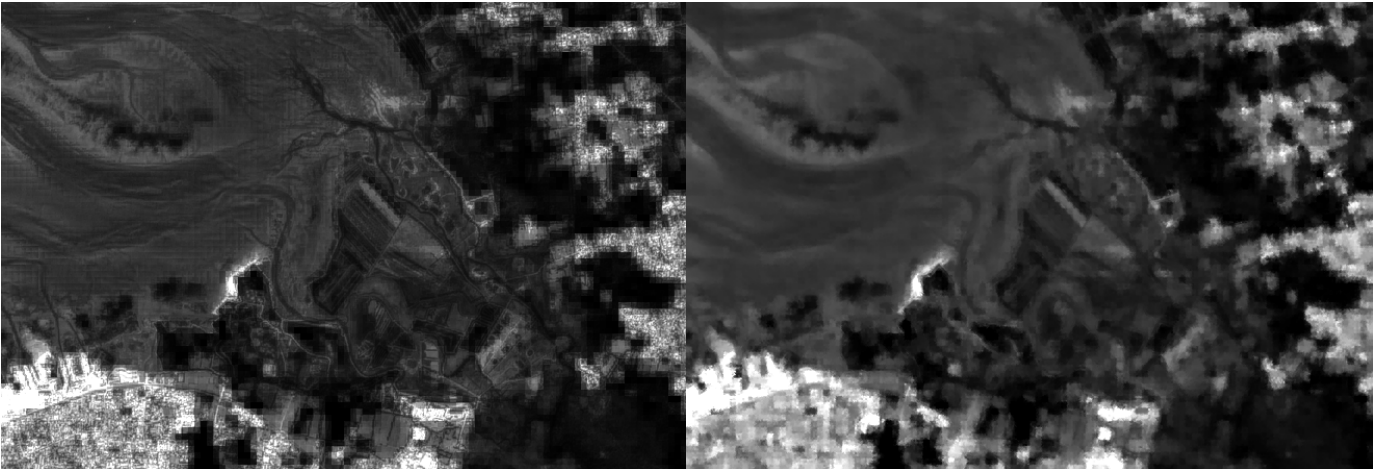


Figure 9. Left: Incorrect image obtained when shared values are not used, that demonstrates their role in maintaining spatial consistency. Right: Incorrect image obtained when only mean square optimisation is applied, that shows the importance of the \bar{q} ratio sharpening step described in Section III-B.



Figure 10. Incorrect image obtained when the estimation of weights W is omitted while maintaining the \bar{q} ratio sharpening step.

CONCLUSION

This article presents a super-resolution method based on exploiting both the local consistency between neighborhood pixels and the geometric consistency of sub-pixel constituents across multispectral bands. Figs. 6, 7, 8 show the result of applying this method on Sentinel-2 images, in order to bring all bands at 20m/pixel and 60m/pixel down to the highest resolution at 10m/pixel. The algorithm is however generic and could be applied to other multi-resolution and multispectral satellite images. Further work could include the usage of secondary images, taken from a satellite with low temporal resolution but with a higher spatial resolution, in order to extract the geometric information used for the super-resolution. Assuming the pixel geometry do no change much between these acquisitions, then Sentinel-2 images could then be enhanced below 10m/pixel. This form of multi-satellite temporal super-resolution would combine high temporal frequency with high spatial resolution. Another trail of research would be to incorporate the super-resolution algorithm directly within the atmospheric correction step [19], rather than applying it as a separate stage. Indeed, using higher-resolution pixels instead of low-resolution bands for calibrating the atmospheric correction may lead to better accuracy.

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SOURCE CODE

The source code for this super-resolution algorithm is provided as Free/Libre software, under either (your choice) the lesser GNU public licence v2.1 or more recent, or the CeCILL-C licence. The library is written and usable directly in C++ and it is also wrapped in a Python package. A ready to use Python script for super-resolving Sentinel-2 images is provided. See <http://nicolas.brodu.net/recherche/superres/>.

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